Spatially Explicit Optimization
Attachment "A" of:
"Incorporation of Wildland Fuels Information Into Landscape Scale Land Use and Planning Processes"

00-G-11221611-045

FINAL PROJECT REPORT

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October 25, 2001

BACKGROUND

The use of mathematical models for managing fires has a rich and varied history in North America. Martell (1982) traces the application of operations research methods in forest fire studies back to the early 1960s. The earliest potential applications of operations research techniques to wildland fire management were mentioned by Shephard and Jewell (1961). Follow-up work by Parks and Jewell (1962) generated considerable interest by examining the use of differential equations and calculus to identify the optimal suppression force for a forest fire. Swersey (1963) and McMasters (1966) extended Parks and Jewell's work by focusing on the optimal mix of different suppression units and the effects of labor constraints on resource allocation rules.

Growing familiarity with optimization techniques spawned additional fire management applications, notably analyses of detection options (Kourtz and O'Regan, 1971) and airtanker retardant delivery systems (Simard, 1979; Greulich and O'Regan, 1982). Fire suppression has continued to receive considerable interest through the use of optimal control theory (Parlar and Vickson, 1982), nonlinear programming (Aneja and Parlar, 1984), and catastrophe theory (Hesseln et al., 1998). In addition to optimization, simulation modeling has provided useful insights for evaluating management alternatives, especially in an uncertain decision environment (Ramachandran, 1988; Fried and Gilless, 1988; Mees and Strauss, 1992; Mees et al., 1993; Gilless and Fried, 1999). Other simulation work oriented towards allocating management resources in fire containment efforts includes Mees (1985), Anderson (1989), and Fried and Fried (1996).

Boychuk and Martell (1988) used Markov chains to analyze seasonal hiring requirements. Martell et al. (1989) modeled seasonal variation in the occurrence of human-caused forest fires. Mills and Bratten (1982) described the use of an economic efficiency system for minimizing the "cost plus net value change" of various fire management alternatives. And, a number of expert systems have also been developed for wildfire containment applications, such as Kourtz (1987), Saveland et al. (1988), Fried and Gilless (1989), Stock et al. (1996), and Hirsch et al. (1998).

INTRODUCTION

Recent interest in fire and fuel management is particularly motivated by the high fuel levels in many forested areas, especially in ecosystems with short fire return intervals and where fire was excluded during the 20th century (Arno and Brown, 1991; Covington and Moore, 1994). These fuel conditions create a need for the long-term reduction of fuel loads. This management problem will require new and spatially explicit management science methods expanded to landscape scales. Most approaches in the current literature boil down to a greedy algorithm selecting areas according to some ranking of risk or effectiveness (e.g., Chew et al., in press; Omi et al., 1981). In this paper, we will explore a few options for attacking this problem that are a bit richer in either dynamic or spatial relationships, and point out the significant difficulties along the way. First, we will look at approaches for creating and maintaining a pre-specified set of forest conditions that are deemed desirable from a fuels management perspective. Second, we will look at capturing the spatial relationships suggested by a fire event targeted for long term fuels management.

CREATING AND MAINTAINING A DESIRED FOREST

A number of authors have suggested that fire and fuels conditions in ponderosa pine and mixed conifer forests would be improved by creating a forest with densities and age structures that emulate historical conditions or natural processes (e.g., Brown et al. 1999, Keifer et al. 2000). Scheduling the removals (through controlled burning, mechanical removals, commercial timber harvests, and other methods) to create and maintain such a forest presents a different emphasis to the forest management scientist from the traditional timber harvest scheduling models that focused on removals to optimize some measure of present net worth. Differences in objectives notwithstanding, traditional timber scheduling approaches may be useful for landscape planning if harvested removals are linked to desired changes in fire behavior.

Traditional Timber Harvest Scheduling Models

Traditional approaches to even-aged timber harvest (and other removals) scheduling are summarized in Johnson and Scheurman (1977). Using the "Model I" as the most typical formulation, the approach defines discrete time periods and limits to the age of harvest, so as to create a finite number of scheduling options on specified units of land. Assuming that the objective function is discounted net revenue maximization, the problem is typically formulated as:

Maximize:

$$\sum_{t=1}^{p}(R-L)/(1+r)^{T_t}H_t - \sum_{i=1}^{m}\sum_{j=1}^{n_i}\sum_{t=1}^{p}C_{ijt}/(1+r)^{T_t}X_{ij}$$

Subject to:

$$\sum_{j=1}^{n_i} X_{ij} \leq A_i \qquad \forall i$$

$$\sum_{i=1}^{m}\sum_{j=1}^{n_i} V_{ijt} X_{ij} = H_t \qquad \forall t$$

Where:

Indexes

t indexes time periods

i indexes analysis areas

j indexes management regimes

p = number of time periods in the planning horizon

m = number of analysis areas

n_i = number of harvesting regimes for analysis area i

Variables

H_t = total harvest in the time period t

X_{ij} = number of acres in analysis area i allocated to management regime j

Parameters

A_i = number of acres in analysis area i

V_{ijt} = the yield volume per acre in analysis area i, in harvesting regime j, and in time period t

T_t = the time (in years) at the midpoint of time period t

r = the discount rate

R = the nominal revenue per unit of volume harvested

L = the nominal cost per unit of volume harvested

C_{ijt} = the nominal per-acre cost in time period t for analysis area i and harvest regime j

Notice that discrete removal regimes with a limited number of management actions are defined with discrete time periods, and yields are approximated accordingly. It should also be noted that this approach is quite conducive to constraints such as nondeclining yield, formulated as:

$$H_t \leq H_{t+1} \qquad t = 1,\ldots,p-1$$

If we were to try to use this approach to create and maintain the desired forest for fuels management purposes, we would probably start by defining our land units at a fairly fine scale (so that they are relatively spatially explicit). Then, state variables (S_{ikt}) would be defined that track the number of trees in different age classes (k) in each land unit (i), and each time period (t). If each management regime (X_{ij}) were defined such that specific-aged trees are removed, then the state variables could be defined as some function of the management regimes:

$$S_{ikt} = f(X_{ij}) \qquad \forall i,j,k,t$$

and the state variables could be used in objective functions or constraints to optimize the creation and maintenance of the desired forest.

The primary problem with this approach is that there is an extremely large number of management regimes implied. For each land unit, a nearly infinite number of different options would be possible in terms of the number of different trees in different age classes harvested in different time periods. In order to even approach an optimal

conversion and maintenance of the desired age structure over all the land units, the choice variables would have to include a reasonable representation of all options available. This problem arises because the Model I (and Model II) formulations are even-aged management models, and the problem of creating a forest with a particular location-specific age distribution is really an "any-aged management" problem.

An Any-Aged Management Approach

The early studies that formulated optimization models for uneven-aged management (e.g., Adams and Ek, 1974) assumed a steady state solution and viewed the stand-level uneven-aged management problem as one of determining the optimal diameter class distribution, the optimal species mix, the optimal cutting cycle length, and also an optimal conversion strategy for stands not initially in the desired steady state (Hann and Bare, 1979; Gove and Fairweather, 1992). The steady state assumptions make the problem more tractable, with choice variables (based on diameter classes) and a cutting cycle that apply across the stand.

The less restrictive approach in Haight et al. (1985) is probably more conducive to fuels management, because it does not necessarily need to be used in the context of a steady state solution. Haight (1987) and Haight and Monserud (1990) subsequently coined the term "any-aged management" which we use here as the underlying basis for fuels management. The problem with the model in Haight et al. is that only relatively small problems can be solved (as it is nonlinear). For this apparent reason, Haight et al. only apply their model to a single stand. What is needed for fuels management is a

formulation that can handle many diverse (and spatially explicit) stands and be solvable with large problems covering large areas of land.

The Haight et al. model defines choice variables that directly relate to the trees removed, rather than the area treated as in the traditional timber harvest scheduling models described above. The Haight et al. model is nonlinear because the periodic ingrowth and mortality of trees in each age class are nonlinear functions of the number of trees in (potentially) all the age classes. One approach to the large-scale fuels management problem might be to relax this assumption enough to formulate a linear program. At this point, let us abandon the economic objective function in favor of one that directly optimizes the creation and maintenance of the desired forest. A possible formulation along these lines might look like:

Minimize:

$$\sum_{i=1}^{m}\sum_{k=1}^{q}\sum_{t=1}^{p}\lambda_{ikt} \tag{1}$$

Subject to:

$$\lambda_{ikt} \geq S_{ikt} - \overline{U}_{ik} \qquad \forall i,k,t \tag{2}$$

$$\lambda_{ikt} \geq \overline{U}_{ik} - S_{ikt} \qquad \forall i,k,t \tag{3}$$

$$S_{ik1} = \overline{S}_{ik} - X_{ik1} \qquad \forall i,k \tag{4}$$

$$S_{i1t} = \sum_{k=1}^{q}(r_{ik}S_{ik(t-1)}) + A_{it} - X_{i1t} \qquad \forall i; t = 2,\ldots,p \tag{5}$$

$$S_{ikt} = (1 - M_{i(k-1)})S_{i(k-1)(t-1)} - X_{ikt} \qquad \forall i; k = 2,\ldots,q-1; t = 2,\ldots,p \tag{6}$$

$$S_{iqt} = (1 - M_{i(q-1)})S_{i(q-1)(t-1)} + (1 - M_{iq})S_{iq(t-1)} - X_{iqt} \qquad \forall i; t = 2,\ldots,p \tag{7}$$

Where:

Indexes

t indexes time periods (there are p of them)

i indexes analysis areas (there are m of them)

k indexes age groups (there are q of them)

Note: t and k are defined with the same time step.

Variables

X_{ikt} = the number of trees harvested in time period t, in area i, and in age class k

S_{ikt} = the number of trees in time period t, in area i, and in age class k,

A_{it} = the number of trees artificially planted in area i in time period t

Parameters

r_{ik} = the natural regeneration rate for age class k in area i

$\overline{U}_{ik}$ = the targeted number of trees in age class k in area i

$\overline{S}_{ik}$ = the initial number of trees in age class k in area i

M_{ik} = the mortality rate for age class k in area i

Equation (1) minimizes the sum of all deviations from the desired forest, over all land units, age classes, and time periods. Equations (2) and (3) define the lambda variables as the absolute value of the deviation of the state variables from the desired forest variables. Equation (4) calculates the state variables for the first time period as the initial conditions less any removals (as choice variables) that take place at the beginning of the first time period. It is assumed here that all removals happen at the beginning of each time period. Removals are also accounted for in Equations (6) and (7). Equations (5)-(7) track the trees in each land unit as they move through the age classes from time

period to time period. Note that the time periods and the age classes need to be defined with consistent time steps. Equation (5) applies to age class 1, Equation (7) applies to the oldest age class, and Equation 6 applies to all age classes in between. Natural regeneration is accounted for as a parameter and artificial planting is accounted for as a choice variable in Equation (5). Mortality is accounted for as a parameter in Equations (6) and (7).

Clearly, the weaknesses of this formulation are the heroic assumptions that mortality and regeneration in each age class and each area are linear functions of the number of trees in that age class and that area, such that M and r are fixed constants. Mortality and regeneration have always been problematic in timber harvest scheduling models because they are affected by so many factors and because they are subject to significant randomness. The impact of these simplifying assumptions would vary from case to case, but it is not hard to imagine situations where they would yield wildly incorrect results. As a simple example, imagine a state where very high densities of young seedlings already crowd the under-story, but Equation (5) would continue to add regeneration at the same rate as if there were no seedlings at all. The analyst might be able to repeatedly solve the model, adjusting the r and M parameters over the planning horizon to closer match the state of the forest in solution (in each time period). Whether such an iterative approach would converge on a stable solution or not would, again, be expected to vary from case to case. It does help that the purpose of the model is to come as close as possible to a pre-specified forest. Still, if the initial state is much different from the desired state (which we would expect when the initial fuels condition is highly undesirable), then the forest mortality and regeneration rates for the desired state would

not be very accurate during the conversion period. The fact of the matter is that the any-aged forest management problem is much more difficult from an optimization modeling standpoint than the even-aged problem, which means that no totally satisfactory large-scale approach is available at this time to optimally create and maintain a pre-specified forest structure that is not even-aged.

SPATIAL RELATIONSHIPS—OPTIMIZATION WITH A TARGET FIRE

The previous approach is spatially-explicit in that choice variables are defined with sufficient spatial detail to emulate historical conditions across the landscape, but no spatial relationships are really included. The obvious source of spatial relationships for fuels management is the spatial nature of fire itself. That is, the spatial layout of fuels and the spatial relationships between fuel loads in different areas are important because they can affect fire behavior. For example, a large conflagration involving vertically-contiguous fuels at the head of a fire front can increase the probability of that fire moving to the crowns. Stand replacement usually results if wind, moisture, and fire intensity are conducive to sustaining such a crown-fire event. In order for a closed-form optimization model to account for these spatial relationships, however, a particular fire event needs to be accepted as the "target" of our fuels management efforts. Otherwise, it is uncertain where the fire of concern starts and what it does from there. If it is truly random where a fire of concern might start and how it might behave, then fuels management may be a spatial problem only in the sense that there might be general spatial guidelines for the desired fuel structure that we wish to obtain (which could be included with the approach discussed in the previous section). If, on the other hand, a particular fire can be accepted

as a target for fuels management, then the placement of fuels management effort might be optimized to account for the implied spatial relationships.

Even if we assume that we can define a particular target fire to guide our fuels management efforts, it is still not completely clear what our specific objective should be. Traditionally, fire managers have focused on fire suppression strategies that emphasize direct control of the fire or containment of its perimeter within pre-determined or natural barriers. When confronted with fires that exceeded control or containment capabilities of available suppression resources, the fall-back position has called for the protection of valued resources (i.e., homes, communities or other human developments). This approach is also relevant when it is decided to let a fire burn so as to restore natural fire processes or as a part of fuels reduction efforts. Recent policy changes (Zimmerman and Bunnell 1998) call for an expansion of strategies for managing fires, especially at the landscape scale. As Finney (2001:219-220) states "Two basic strategies for landscape-level fuel management are to contain fires and to modify fire behavior...a spatial arrangement of treatments that primarily modifies fire behavior would involve area-based or dispersed patterns...For fire modification, it is clear that the greatest reduction in fire size and severity occurs when fuel treatment units limit fire spread in the heading direction." One option between letting a fire burn unhindered and attempting suppression is thus to slow its spread across the landscape, relative to any valued resources that it threatens.

Let us assume that there are distinct areas of concern (such as towns, summer homes, campgrounds, ski areas, and so forth), and that a fire management objective is to delay ignition of those "protection areas" in the target fire event as long as possible. The

advantages of such a delay would include: (1) maximizing the chances that other suppression efforts or independent factors such as weather changes might cause the fire to subside before the protection areas are impacted; and (2) maximizing the time available for building fire line around the protection areas, for modifying fuels to reduce a fire's severity near the protection areas, or for evacuation of the protection areas.

If the objective of long-term fuels management is to mitigate the effects of a particular target fire, with known origin(s) and spread behavior, then one approach (from Hof et al., 2000) could be as follows. To begin, the landscape would be defined with a grid of cells to capture spatial location. The management variables would need to be defined as application of fuels reduction efforts in each cell (such as prescribed burning, mechanical removals, and so forth) and these efforts are to be scheduled over a fairly long period of time because only so much fuels reduction can be accomplished in a given year (or season). Thus, discrete time periods of, say, one to ten years would be defined and indexed with t. The trajectory of each cell's fuel load over time, and its response to fuels management, would have to be tracked as well. Such a model might be formulated as:

Maximize: λ (8)

Subject to:

$$\lambda \leq T^{\circ}_{mnt} \quad \forall\, t \qquad (9)$$

$$T^{\circ}_{abt} = 0 \quad \forall\, t \qquad (10)$$

$$T^{\circ}_{ijt} \leq T'_{hkt} \quad \begin{array}{c} \forall\ i, j, t \\ \forall\, (h, k) \in \Omega_{ij} \end{array} \qquad (11)$$

$$T'_{ijt} - T^{\circ}_{ijt} = f_{ij}(F_{ijt}) \quad \forall\, i, j, t \qquad (12)$$

$$F_{ijt} = \sum_{k=1}^{K_{ij}} B_{ijkt}\ X_{ijk} \qquad \forall\, i, j, t \qquad (13)$$

$$\sum_{k=1}^{K_{ij}} X_{ijk} \leq 1 \qquad \forall\, i, j \qquad (14)$$

$$\sum_{i} \sum_{j} \sum_{k}^{K_{ij}} D_{ijkt} X_{ijk} \leq \overline{X}_t \qquad \forall\, t \qquad (15)$$

$$0 \leq X_{ijk} \leq 1 \qquad \forall\, i, j, k \qquad (16)$$

Where:

<u>Indexes</u>

i indexes cell rows, as does h

j indexes cell columns, as does k

k indexes management prescriptions (there are K_{ij} of them for cell ij)

<u>Variables</u>

T°_{ijt} = the time that the target fire front ignites cell ij, if it occurs in time period t

T'_{ijt} = the time that the target fire front leaves cell ij, if it occurs in time period t

T'_{hkt} = the time that the target fire front leaves cell hk (and potentially ignites cell ij) if it occurs in time period t

F_{ijt} = the fuel available for combustion in cell ij and in time period t

X_{ijk} = the proportion of cell ij allocated to management prescription k

f_{ij} =an empirical function that relates available fuels in cell ij to the duration of time between entry and exit of the fire front

<u>Parameters</u>

a,b = the row and column of the fire origin cell

m,n = the row and column of the protected area cell

Ω_{ij} = the set of row and column indexes for cells that can potentially ignite cell ij. Note: this would typically be some subset of the cells adjacent to cell ij and would be determined primarily by a combination of wind conditions during the target fire and topography.

B_{ijkt} = the available fuel in time period t, in cell ij, that results if management prescription k is applied

D_{ijkt} = a dummy parameter that is equal to 1 if management prescription k applies fuels reduction in time period t, in cell ij, and is zero otherwise

$\overline{X}_t$ = the total number of cells that can be treated with fuels reduction in time period t

The objective function (8) together with the inequalities in (9) maximize the minimum ignition time of the protection cell *mn* across time periods (the target fire might occur in any time period). Other objective functions that aggregate time periods would also be possible. Equation (10) sets the ignition time of the origin cell to zero. Equation (11) relates the ignition times of each cell to the times that the fire front leaves the cells which can potentially ignite it. The multiple inequalities in (11) cause each cell to be ignited by the first potentially-igniting cell that the fire front departs from. Equation (12) relates the duration of time that it takes the fire front to move through each cell to the available fuel in that cell, given existing weather and topography. In practice, the spread rate functions (f) would be estimated with fire prediction models or empirical data. Linear approximation of this f function is discussed in Hof et al. (2000). Constraints (10) - (12) account for a potential target fire in each time period. Constraint (13) applies the management variables (that include scheduling) to determine the changing available fuel

load in each cell *ij* over time. Constraint (14) restricts the sum of the management prescription variables to be less than or equal to one for each cell, and constraint (15) limits the amount of fuels management treatment that can be applied in each time period. It may be desirable to replace (16) with binary integer constraints on all X_{ijk}, to force either complete treatment or no treatment of each cell in any given time period. As the formulation stands, it is assumed that the B_{ijkt} parameter is applicable to fractional values of X_{ijk}. Extension of this approach to multiple protection areas is straightforward (see Hof et al., 2000).

This model, again, is based on using a target fire to guide long term fuels management. The approach is similar to the use of particular storm events (such as a 500-year flood) to guide watershed and flood control planning. Presupposing highly random fire behavior conditions such as wind speed and direction or location of lightning strikes, however, may be far less certain than the path of water flow on a landscape. If a different fire event eventually occurs, the fuels management strategy based on the target fire may or may not be desirable. At any rate, the model is readily solvable if the *f* functions are linear, because it is then a linear program with continuous variables.

CONCLUSION

Overall, long term fuels management presents a formidable problem for management scientists. Treating the problem as one of creating and maintaining a particular forest (which is believed to be desirable from a fuels perspective either because of historical conditions or some other criterion) is difficult because it is an any-aged forest management problem that is intrinsically nonlinear. The assumptions necessary to

make such a problem linear are rather heroic. Treating the problem so as to account for the spatial nature of fire itself is difficult because fire origins and behavior can be quite random and unpredictable. It is necessary in a closed form model to accept a particular fire event as the target for fuels management. An approach that focuses on spatial fuel pattern, per se, might show promise, but guidelines for desirable patterns are not apparent. Monte Carlo approaches that simulate many fires might show promise in accounting for the uncertainty of fire origin and behavior, but heuristics for finding near-optimal solutions have yet to be developed and the basic computing time necessary to adequately simulate an adequate number of fires may be prohibitive. Clearly, much additional work is needed on all aspects of the spatial and dynamic management of fuels at the landscape scale.

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